DCMT – 401

IV Semester B.A./B.Sc. Examination, September/October 2023
(NEP – Freshers)
MATHEMATICS (Paper – IV)
Partial Differential Equations and Integral Transforms

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer **all** the Parts.*

PART – A

I. Answer **any six** of the following. **(6×2=12)**

1) Form a partial differential equation by eliminating arbitrary constants from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

2) Solve : $\sqrt{p} + \sqrt{q} = 1$.

3) Solve : $[D^2 + 3D\,D' + 2\,(D')^2]z = 0$.

4) Write the condition for partial differential equation of second order to be elliptic and give an example for elliptic partial differential equation.

5) Find L{sin5t cos3t}.

6) Find $L^{-1}\left\{\frac{4}{(s+3)^5}\right\}$.

7) Calculate 'a_0' in the Fourier series of $f(x) = e^x$ in (0, 2).

8) Obtain half range sine series of f(x) = x in (0, π).

PART – B

II. Answer **any three** of the following. **(3×4=12)**

1) Form the partial differential equation by eliminating arbitrary functions from z = f(x + ay) + g(x – ay).

2) Solve : $\left(\frac{y^2 z}{x}\right)p + xzq = y^2$.

P.T.O.

3) Solve : $\frac{\partial^3 z}{\partial x^2 \partial y} = \sin(2x + 3y)$ by direct integration method.

4) Solve by Charpit's method, $pxy + pq + qy - yz = 0$.

5) Solve : $9(p^2 z + q^2) = 4$.

PART – C

III. Answer **any three** of the following. **(3×4=12)**

1) Solve : $r - 2s + t = e^{x + 2y}$.

2) Solve : $(D^2 - DD')z = \cos x \cos 2y$.

3) Solve : $(D - 3D' - 2)^2 z = 2e^{2x} \tan(y + 3x)$.

4) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

5) Solve $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions,

i) $u(0, t) = 0, u(1, t) = 0, \forall t$

ii) $u(x, 0) = x^2 - x, 0 \le x \le 1$.

PART – D

IV. Answer **any three** of the following. **(3×4=12)**

1) Find $L\{e^{3t} \sin 5t \sin 3t\}$.

2) Find $L\{f(t)\}$, if $f(t) = \begin{cases} -1 & 0 \le t \le 4 \\ 1 & t > 4 \end{cases}$.

3) Find $L^{-1}\left\{\frac{1}{(s+1)(s+2)(s+3)}\right\}$.

4) Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$.

5) Solve by using Laplace transform

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, \; y(0) = 0, \; y'(0) = 0.$$

PART – E

V. Answer **any three** of the following. **(3×4=12)**

1) Obtain the Fourier series for the function $f(x) = x - x^2$ over the interval $-\pi \leq x \leq \pi$.

2) Obtain the Fourier series expansion of the function f(x) defined as
$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}.$$

3) Obtain the half-range cosine Fourier series of $f(x) = x^2$ in $0 < x < \pi$.

4) Find the Fourier transform of the function $f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$.

5) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$.