IV Semester B.Sc. Examination, September/October 2023
(CBCS) (Repeaters) (2015-16 Onwards)
MATHEMATICS
Mathematics – IV

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

I. Answer **any five** questions. (5×2=10)

1) a) Define normal subgroup of a group.

b) If $f : G \longrightarrow G'$ is a homomorphism, then prove that $f(e) = e'$, where e and e′ are the identity elements of G and G′ respectively.

c) Calculate a_0 in the Fourier series of $f(x) = e^x$ in $(-\pi, \pi)$.

d) State Taylor's theorem for a functions of 2 variables.

e) Find $L[t^3 - 4t^2]$.

f) Find $L^{-1}\left[\dfrac{S-1}{(S-1)^2+9}\right]$.

g) Find particular integral of $(D^2 + 4D + 4)y = e^{2x}$.

h) Verify whether $(1 - x^2)\,y'' - 3xy' - y = 0$ is exact.

PART – B

II. Answer **one full** question. (1×15=15)

2) a) Prove that a subgroup H of a group G is normal subgroup of G iff $gHg^{-1} = H, \forall g \in G$.

b) Prove that the centre of a group G is a normal subgroup of G.

c) If $f : G \longrightarrow G'$ is a homomorphism, then prove that f(G) is a subgroup of G′.

OR

3) a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

b) State and prove Cayley's theorem.

c) State and prove fundamental theorem of homomorphism.

PART – C

III. Answer **two full** questions. **(2×15=30)**

4) a) Obtain the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.

b) Obtain half range cosine series of $f(x) = \sin x$, $0 < x < \pi$.

c) Expand $x^2y + 3y - 2$ in powers of x and y by Taylor's series upto 2^{nd} degree terms.

OR

5) a) Find the extreme values of the function $f(x) = xy(1 - x - y)$.

b) Expand $f(x) = (x - 1)^2$ in $0 < x < 1$ in terms of half-range sine series.

c) Find the maximum and minimum distances of the point (1, 2, 3) from the sphere $x^2 + y^2 + z^2 = 56$ using Lagrange's method.

6) a) Find $L[e^t \sin 2t]$ and $L[t \sin t]$.

b) Evaluate

i) $L[t^3 + e^{2t} + \cos 2t]$

ii) $L[\cos^2 t]$.

c) Find $L^{-1}\left[\frac{1}{(S+2)(S+4)}\right]$.

OR

7) a) Find

i) $L[t^3 e^{-3t}]$

ii) $L[e^{-t}(2\cos 5t - 3\sin 5t)]$.

b) Find $L[\cos 2t * \sin 3t]$.

c) Find $L^{-1}\left[\frac{S+3}{(S+3)^2+36}\right]$.

PART – D

IV. Answer **one full** question. **(1×15=15)**

8) a) Solve $(D^2 + 2D + 4)y = e^{2x}$.

b) Solve $4x^2y'' + 4xy' - y = 4x^2$.

c) Solve $(D^2 - 6D + 9)y = 3e^{-4x}$ given that e^x is a part of the complementary function.

OR

9) a) Solve $(D^2 - 5D + 6)y = e^{4x} + \sin 2x$.

b) Solve $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0$.

c) Solve $\frac{d^2y}{dx^2} + 9y = \sec 3x$ by the method of variation of parameters.
