III Semester B.Sc. Examination, April/May 2023
(NEP Scheme)
MATHEMATICS
Paper – III : Ordinary Differential Equations and Real Analysis – I

Time : 2½ Hours

Max. Marks : 60

Instruction : *Answer **all** questions.*

PART – A

I. Answer **any six** of the following :   (6×2=12)

1) Show that $(x^2 - ay)dx + (y^2 - ax)dy = 0$ is exact.

2) Find the general solution of $y = px + \dfrac{a}{p}$.

3) Solve : $\dfrac{d^2y}{dx^2} - 16y = 0$.

4) Find the particular integral of $(D^2 + 4D + 4)y = e^{2x}$.

5) Define a convergent sequence with an example.

6) Show that $\left\{\dfrac{1}{n}\right\}$ is a monotonically decreasing sequence.

7) State D'Alembert's ratio test for the series of positive terms.

8) Test the convergence of the series $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots \infty$.

PART – B

II. Answer **any three** of the following :   (3×4=12)

1) Verify for exactness and solve :

$(4x + 3y + 1) \, dx + (3x + 2y + 1) \, dy = 0$.

P.T.O.

2) Solve : $y = 2px + y^2p^3$.

3) Solve : $y = x + p^3$.

4) Find the general and singular solutions of $\sin px.\cos y - \cos px.\sin y = p$.

5) Find the orthogonal trajectories to the curve $r = a(1 - \cos\theta)$ where 'a' is a parameter.

PART – C

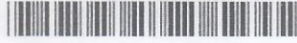III. Answer **any three** of the following :   (3×4=12)

1) Solve : $(D^2 - 5D + 6)y = e^{4x} + \sin 2x$.

2) Solve : $x^2y'' - xy' + 2y = x \log x$.

3) Solve $xy'' - (1 + x)y' + y = 0$ given that, $(x + 1)$ is a part of complementary function.

4) Solve : $y'' + y = \sec x$ by the method of variation of parameters.

5) Verify the condition for integrability and solve,

$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx) dz = 0$.

PART – D

IV. Answer **any three** of the following :   (3×4=12)

1) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty} (a_n + b_n) = a + b$.

2) Prove that a monotonic increasing sequence which is bounded above is convergent.

3) Find the limit of the sequence $\{0.3, 0.33, 0.333, ... \}$.

P.T.O.

4) Examine the convergence of the sequence

   i) $\{n[\log(n+1) - \log n]\}$

   ii) $\left\{\dfrac{n+1}{n}\right\}$.

5) Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.

PART – E

V. Answer **any three** of the following : (3×4=12)

1) State and prove D'Alembert's ratio test for the series of positive terms.

2) Test the convergence of the series :

   $$1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \ldots$$

3) Discuss the convergence of the series :

   $$\sum_{n=1}^{\infty} \frac{\left[(n+1)x\right]^n}{n^{(n+1)}}.$$

4) Test the convergence of the series

   $$\sum_{n=1}^{\infty} \frac{1.2.3.\ldots n}{3.5.7.\ldots(2n+1)}.$$

5) Find the sum to infinity of the series

   $$\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \ldots$$