61506

V Semester B.A./B.Sc. Examination, April/May 2023
(CBCS Scheme)
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** the questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Write the Euler's equation when is independent of x.

b) Find the differential equation in which functional $I = \int_{x_1}^{x_2} \left[1 + xy' + x(y')^2\right] dx$ has an extremum.

c) Define Geodesics on surface.

d) Evaluate $\int_C (5xy\,dx + y^2 dy)$ where C is the curve in the xy-plane $y = 2x^2$ from (0, 0) to (1, 2).

e) Evaluate $\int_1^2 \int_3^4 (xy + e^y)\, dy\, dx$.

f) Write the cylindrical polar co-ordinate in triple integral.

g) State Gauss Divergence Theorem.

h) If V is the volume of a region bounded by a closed surface S. Show that $\iint_S \nabla r^2 . \hat{n} ds = 6V$ by using Gauss Divergence theorem.

PART – B

Answer **any two full** questions : **(2×10=20)**

2. a) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

b) Solve the variational problem $I = \int_0^{\pi/2} \left(y^2 - (y')^2\right) dx = 0$ given $y(0) = 0$, $y\left(\pi/2\right) = 2$.

OR

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3. a) Find the function y which makes the integral $\int_{x_1}^{x_2} \left(y^2 + 4(y')^2\right) dx = 0$ an extremum.

   b) Find the curve on which the functional $I = \int_0^1 \left[(y')^2 + 12xy\right] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremed.

4. a) Find the geodesic on a right circular cylinder.

   b) A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary.

OR

5. a) Show that the extremal of the functional $\int_0^2 \sqrt{1+(y')^2}\, dx$ subject to the constraint $\int_0^2 y dx = \pi/2$ and end conditions $y(0) = 0$, $y(2) = 0$ is a circular arc.

   b) Show that the functional $\int_{x_1}^{x_2} \left(y^2 + x^2 y^1\right) dx$ assumes extreme values on the straight line $y = x$.

PART – C

Answer **any 2 full** questions : **(2×10=20)**

6. a) Evaluate $\int_C (x + 2y)\, dx + (4 - 2x)\, dy$ around the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ in the counter clockwise.

   b) Evaluate $\iint_R xy\, dx\, dy$ where R is the first quadrant of the circle $x^2 + y^2 = a^2$ where $x_1 y \geq 0$.

OR

7. a) Evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy dx$ by changing the order of integration.

   b) Find the volume generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

8. a) Evaluate $\iint\limits_R y\, dx\, dy$ where R is region bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

   b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$.

OR

9. a) If R is the region bounded by the planes x = 0, y = 0, z = 0, z = 1 and the cylinder $x^2 + y^2 = 1$. Evaluate $\iiint_R xyz\, dx\, dy\, dz$ by changing it to cylindrical polar co-ordinates.

b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

PART – D

Answer **any 2 full** questions : **(2×10=20)**

10. a) State and prove Green's theorem.

b) Using Gauss Divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes x = 0, y = 0, z = 0, x = 1, y = 2, z = 3.

OR

11. a) Evaluate by Stoke's theorem $\oint_C yzdx + zxdy + xydz$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

b) Verify Green's theorem for $\vec{F} = (x^2 - y^2)\,\hat{i} + 2xy\hat{j}$ over the rectangular region bounded by the lines x = 0, y = 0, x = a, y = b.

12. a) State and prove Gauss Divergence theorem.

b) Using Green's theorem evaluate $\int_C e^{-x} \sin y\, dx + e^{-x} \cos y\, dy$ where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$.

OR

13. a) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

b) Evaluate by stoke's theorem $\oint_C \sin z\, dx - \cos x\, dy + \sin y\, dz$ where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.
