61505

V Semester B.A./B.Sc. Examination, April/May 2023
(CBCS Scheme) (F+R)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

1. Answer **any five** questions. (5×2=10)

a) In a ring (R, +, ·) prove that $a.(b-c) = a.b - a.c, \forall a, b, c \in R$.

b) Define subring of a ring. Give an example.

c) Define homomorphism of rings.

d) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at (1, 2, –1).

e) Show that the vector $\vec{F} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$ is solenoidal.

f) Prove that $\nabla = 1 - E^{-1}$.

g) Write Lagrange's interpolation formula for unequal intervals.

h) Evaluate $\int_0^3 \frac{dx}{1+x}$ using Simpson's $\frac{3}{8}^{th}$ rule given

x	0	0.5	1	1.5	2	2.5	3
y	1	0.667	0.5	0.4	0.333	0.286	0.25

PART – B

Answer **two full** questions. (2×10=20)

2. a) Prove that $(z_6, +_6, \times_6)$ is a commutative ring w.r.t. $+_6$ and $\times_6$ as two compositions.

b) Prove that the intersection of any two subrings is a subring. Give an example to show that the union of two subrings of a ring need not be a subring.

OR

P.T.O.

3. a) Prove that every field is an integral domain.

   b) Find all the principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. $+_6$ and $\times_6$.

4. a) If $f : R \to R'$ be a homomorphism of R into R′ then show that Ker f is an ideal of R.

   b) State and prove fundamental theorem of homomorphism of rings.

OR

5. a) Prove that an ideal 5 of the ring of integers (z, +, •) is maximal if and only if 5 is generated by some prime integer.

   b) If $f : R \to R'$ be a homomorphism of a ring R into ring R′ then prove that

      i) $f(0) = 0'$, where 0 and 0′ are the zero elements of R and R′ respectively.

      ii) $f(-a) = -f(a), \forall a \in R$.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

   b) Show that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point (1, –1, 2).

OR

7. a) If n is a non-zero constant, then show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. Deduce that when $r \neq 0$, r^n is harmonic if $n = -1$.

   b) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

8. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function then prove that $\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + (\text{grad}\varphi).\vec{F}$.

   b) If $\vec{F} = \nabla(2x^3y^2z^4)$ then prove that $\nabla \times \vec{F} = \vec{0}$.

OR

9. a) If $\phi = xyz$ and $\vec{F} = x^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$ find $\text{div}(\phi\vec{F})$.

   b) If the vector $\vec{F} = (ax + 3y + 4z)\ \hat{i} + (x - 2y + 3z)\ \hat{j} + (3x + 2y - z)\hat{k}$ is Solenoidal then find 'a'.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) By the method of separation of symbols prove that

$$u_0 - u_1 + u_3 - u_4 + ... = \frac{u_0}{2} - \frac{\Delta u_0}{4} + \frac{\Delta^2 u_0}{8} - \frac{\Delta^3 u_0}{16} + ...$$

b) Estimate f(4.2) from the table

x	0	2	4	6
f(x)	2	10	66	218

OR

11. a) Find the cubic polynomial which takes the following values

x	0	1	2	3
f(x)	1	2	1	10

b) Obtain the function whose first difference is $3x^2 + 9x + 4$.

12. a) Using Lagrange's interpolation formula find f(6) from the following data

x	3	7	9	10
f(x)	168	120	72	63

b) Find the value of $\int_1^5 \log_{10} x\, dx$ taking 8 subintervals by Trapezoidal rule.

OR

13. a) Using Newton's divided difference formula find f(18) from the following table

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

b) Evaluate $\int_0^1 e^x dx$ using Simpson's $\frac{3}{8}^{th}$ rule.
