DCMT – 101

I Semester B.A./B.Sc. Examination, May/June 2022
(NEP Scheme)
MATHEMATICS
Algebra – I and Calculus – I

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer **all** the Parts.*

PART – A

I. Answer **any six** of the following : **(6×2=12)**

1) Define the rank of a matrix.

2) Find whether the system of equations has a non-trivial solution or not.

$x + y = 0; x - y - z = 0; 3x + y - z = 0$

3) Check whether limit exists or not for $f(x) = \frac{x}{|x|}$

4) State Cauchy's mean value theorem.

5) Give the expressions for polar sub-tangent and polar sub-normal.

6) Find $\frac{ds}{dx}$ for the curve $y = a \log \sec\left(\frac{x}{a}\right)$.

7) Find the Asymptotes parallel to the co-ordinate axes to the curve $x^2y^2 - a^2x^2 = a^2y^2$.

8) Find the n^{th} derivative of $y = e^{mx}$.

PART – B

II. Answer **any three** of the following : **(3×4=12)**

1) Find the rank of the Matrix 'A' by reducing to row reduced Echelon form

$$A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}.$$

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2) Show that the following system of equations are consistent and hence solve

$x + y + z = 1$

$x + 2y + 3z = 4$

$x + 3y + 5z = 7$

$x + 4y + 7z = 10$

3) Prove that the rank of the transpose of a matrix is same as that of the original matrix.

4) Find the Eigen values and the Eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

5) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley – Hamilton theorem.

PART – C

III. Answer **any three** of the following : **(3×4=12)**

1) Prove that a function which is continuous in [a, b] attains its bounds atleast once in [a, b].

2) Verify Rolle's theorem for the function $f(x) = 8x - x^2$ in [2, 6].

3) State and prove Lagrange's mean value theorem for continuous functions.

4) Expand $f(x) = \log(1 + \sin x)$ upto the terms containing x^4 using Maclaurin's series.

5) Evaluate : $\lim\limits_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{x \tan x}\right)$.

PART – D

IV. Answer **any three** of the following : **(3×4=12)**

1) Show that the angle between the radius vector and the tangent to the curve is $\tan\phi = r\frac{d\theta}{dr}$.

2) Show that the curves $r^m = a^m \cos(m\theta)$ and $r^m = a^m \sin(m\theta)$ intersect orthogonally.

3) Find the Pedal equation of the curve $r = a(1 - \cos\theta)$.

4) Find the radius of curvature of the curve $y = a\cosh\left(\frac{x}{a}\right)$.

5) Find the center of curvature for the curve $y^2 = 4ax$ at (a, a).

PART – E

V. Answer **any three** of the following : **(3×4=12)**

1) Find the n^{th} derivative of the function $y = \frac{(2x-1)}{(x-2)(x+1)}$.

2) State and prove Leibnitz theorem to find the n^{th} derivative of product of two functions.

3) If $y = \tan^{-1}x$, then show that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

4) Determine the position and nature of the double points of the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$.

5) Trace the curve Astroid $x^{2/3} + y^{2/3} = a^{2/3}$, $(a > 0)$.

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