

I Semester B.A/B.Sc. Examination, April/May 2023
(CBCS Scheme) (Repeaters)
MATHEMATICS – I

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

Answer any five questions.

(5×2=10)

1. a) Find the eigenvalues of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.
- b) Does the following system of equations have a non-trivial solutions ?

$$x + 3y + 5z = 0$$

$$x + 6y + 6z = 0$$

$$3x + 9y + 15z = 0$$
- c) Find the n^{th} derivative of $\cos 3x$.
- d) If $z = x^3 - 3xy^2$, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
- e) Evaluate $\int_0^{\pi/2} \sin^7 x \, dx$.
- f) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$.
- g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + z + 5 = 0$.
- h) If the two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find k .





PART – B

Answer **one full** question.

(1×15=15)

2. a) Find the rank of the matrix.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{pmatrix}$$

by reducing to row reduced echelon form.

- b) Find the non-trivial solution of the system of equations $2x - y + 3z = 0$, $3x + 2y + z = 0$, $x - 4y + 5z = 0$.
- c) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

OR

3. a) Reduce the matrix $\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{pmatrix}$ into normal form and hence find the rank.

- b) Show that the following system of equations are consistent and solve them.

$$x + 2y + 2z = 1$$

$$2x + y + z = 2$$

$$3x + 2y + 2z = 3$$

$$y + z = 0.$$

- c) Verify Cayley-Hamilton theorem for the matrix
- $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$
- .

PART – C

Answer **two full** questions.

(2×15=30)

4. a) Find the
- n^{th}
- derivative of
- $\frac{x+3}{(x-1)(x+2)}$
- .

- b) Find the
- n^{th}
- derivative of

i) $\log(x+4)$

ii) $\sin 5x \sin x$.

- c) If
- $y = (\sin^{-1}x)^2$
- , then prove that
- $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$
- .

OR





9. a) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{3} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.
- b) Find the equation of the right circular cone with vertex at the origin, the axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi vertical angle of 30° .
- c) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.

