

## I Semester B.A/B.Sc. Examination, April/May 2023 (CBCS Scheme) (Repeaters) MATHEMATICS – I

Time: 3 Hours

Max. Marks: 70

Instruction: Answer all questions.

PART - A

Answer any five questions.

 $(5 \times 2 = 10)$ 

- 1. a) Find the eigenvalues of the matrix  $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ .
  - b) Does the following system of equations have a non-trivial solutions?

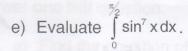
$$x + 3y + 5z = 0$$

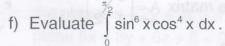
$$x + 6y + 6z = 0$$

$$3x + 9y + 15z = 0$$

c) Find the nth derivative of cos3x.

d) If 
$$z = x^3 - 3xy^2$$
, then prove that  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ .





g) Find the angle between the line 
$$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$$
 and the plane  $x+y+z+5=0$ .

h) If the two spheres 
$$x^2 + y^2 + z^2 + 6z - k = 0$$
 and  $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$  cuts orthogonally, find k.





## PART - B

Answer one full question.

(1×15=15)

2. a) Find the rank of the matrix.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{pmatrix}$$

by reducing to row reduced echelon form.

- b) Find the non-trivial solution of the system of equations 2x y + 3z = 0, 3x + 2y + z = 0, x 4y + 5z = 0.
- c) Find the eigenvalues and eigenvectors of the matrix  $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

OR

- 3. a) Reduce the matrix  $\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{pmatrix}$  into normal form and hence find the
  - b) Show that the following system of equations are consistent and solve them.

$$x + 2y + 2z = 1$$
  
 $2x + y + z = 2$   
 $3x + 2y + 2z = 3$   
 $y + z = 0$ .

c) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ .

Answer two full questions.



- 4. a) Find the n<sup>th</sup> derivative of  $\frac{x+3}{(x-1)(x+2)}$ .
  - b) Find the nth derivative of

i) 
$$\log (x + 4)$$

- ii) sin5x sinx.
- c) If  $y = (\sin^{-1}x)^2$ , then prove that  $(1 x^2)y_{n+2} (2n + 1) xy_{n+1} n^2y_n = 0$ .

OR



- 9. a) Find the shortest distance between the lines  $\frac{x-3}{3} = \frac{y-8}{3} = \frac{z-3}{1}$  and  $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ .
  - b) Find the equation of the right circular cone with vertex at the origin, the axis is the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and which has semi vertical angle of 30°.
  - c) Find the equation of the right circular cylinder of radius 2 and whose axis is the line  $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$ .

Obtain the reduction formula for iten xdx, where n is a positive it

