First Semester B.C.A. Degree Examination, April/May 2023
(CBCS) (Repeaters)
COMPUTER SCIENCE (Paper – I)
Discrete Mathematics

Time : 3 Hours   Max. Marks : 100

***Instruction** : Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. If A = {2, 3, 4, 5} and B = {0, 1, 2, 3} find $A \cap B$.
2. If P = {1, 2} form the $P \times P \times P$.
3. Define Tautology.
4. Define square matrix with an example.
5. If $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 3 \\ 4 & -3 \end{bmatrix}$, find A + 3B.
6. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$.
7. Prove that $\log_b a . \log_c b . \log_a c = 1$.
8. Find ‘n’ if $2({}^nP_3) = {}^nP_5$.
9. Define a group.
10. If $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$, find $|2\vec{a} + \vec{b}|$.
11. Find the distance between the points A(2, – 3) and B(4, 5).
12. Define slope of a line.

SECTION – B

Answer **any six** of the following : **(6×5=30)**

13. Verify whether $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

14. Prove that $\sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$.

15. Consider $f : R \to R$ given by $f(x) = 4x + 5$. Prove that f is one-one and on-to.

16. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

17. Write converse, inverse and contrapositive of the conditional,

    "If I work hard then I get a grade".

18. Solve using Cramer's rule.

    $4x + y = 7;\ 3y + 4z = 5;\ 3z + 5x = 2.$

19. Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

20. If $A = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}$ prove that $(AB)' = B'A'$.

SECTION – C

Answer **any six** of the following : **(6×5=30)**

21. If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$, show that a = b.

22. Prove that the set G = {1, –1, i, – i} is a group under multiplication.

23. Prove that H = {0, 2, 4} is a subgroup of the group G = {0, 1, 2, 3, 4, 5} under addition modulo 6.

24. How many different words can be formed with the letters of the word "MISSISSIPPI" ?

25. If $^{2n}C_3 : {}^{n}C_2 = 44 : 3$, find n.

26. Find the value of λ for which the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other.

27. Show that the points A(1, 2, 3), B(2, 3, 1) and C(3, 1, 2) are vertices of an equilateral triangle.

28. Find the area of the triangle whose vertices are A(1, 3, 2), B(2, –1, 1), C(–1, 2, 3).

SECTION – D

Answer **any four** of the following : **(4×5=20)**

29. Show that the points (2, – 3), (6, 5), (– 2, 1) and (– 6, –7) form a rhombus.

30. Find the area of the triangle whose vertices are (3, 4), (2, –1) and (4, – 6).

31. Find the equation of the locus of the point which moves such that it is equidistant from the points (1, 2) and (– 2, 3).

32. Show that the line joining the points (2, – 3) and (– 5, 1) is parallel to the line joining the points (7, –1) and (0, 3).

33. Find the equation of the line passing through (–2, 2) and the sum of the intercepts on the co-ordinate axes is 3.

34. Find the equations of the line for which

    a) $p = 4, \alpha = 120°$.

    b) $p = 7, \alpha = 60°$.
