61223

# Second Semester B.A./B.Sc. Degree Examination, May/June 2019

*(CBCS – Freshers + Repeaters – 2014-15 and onwards)*

## Mathematics

## Paper II – MATHEMATICS - II

*Time : 3 Hours]* *[Max. Marks : 70*

*Instructions to Candidates : Answer all Parts.*

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) On the set $z$, $*$ is defined by $a * b = a + b - 1, \ \forall \ a, b \in z.$ Find the identity element.

(b) If in a group $G$, $(ab)^2 = a^2 b^2 \ \forall a, b \in G$, then prove that $G$ is abelian.

(c) Find $\phi$ for the curve $r = a \ e^{\theta \cot \alpha}$.

(d) With usual notation prove that $P = r \sin\phi$.

(e) Find the polar subtangent for the curve $r = a(1 - \cos\theta)$.

(f) Find $\dfrac{ds}{dx}$ for the curve $ay^2 = x^3$.

(g) Verify the exactness of the equation $(e^y + 1) \cos x dx + e^y \sin x \, dy = 0$.

(h) Find the general solution for the equation $y = px + \log p$.

## PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) If $Q^+$ is the set of all positive rationals, prove that $(Q^+, *)$ is an abelian group where * is defined by $a * b = \frac{2ab}{3}$

(b) Prove that the fourth roots of unity form an abelian group under multiplication.

(c) Prove that a non-empty subset $H$ of a group $(G, *)$ is a subgroup of $G$ if and only if

$a * b^{-1} \in H, \forall a, b \in H$

Or

3. (a) Prove that the inverse of an element in a group is unique.

(b) Prove that $H = \{0, 2, 4\}$ is a subgroup of a group $G = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6.

(c) Find :

(i) $f \circ g$

(ii) $g \circ f$, for the set $A = \{1, 2, 3\}$ where $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$.

## PART – C

Answer any **TWO** full questions : **(2 × 15 = 30)**

4. (a) With usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$ for the polar curve, $r = f(\theta)$.

(b) Show that the curves

$r^n = a^n \cos n\theta$, $r^n = b^n \sin n\theta$ intersect orthogonally.

(c) Find the radius of curvature of the curve $XY = C^2$.

Or

5. (a) Derive the formula for Radius of curvature in Cartesian form.

(b) Find the angle of intersection of the curves $r = a(1 - \cos\theta)$, $r = 2a\cos\theta$.

(c) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where $a$ and $b$ are connected by the relation $ab = c^2$.

6. (a) Find all the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$.

(b) Find the position and nature of the double points of the curve

$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$

(c) Find the area bounded by the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or

7. (a) Find the pedal equation of the curve $y^2 = 4a(x + a)$.

(b) Find the perimeter of the cardioid $r = a(1 + \cos\theta)$.

(c) Find the volume of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the $x$ – axis.

PART – D

Answer any **ONE** full question : **(1 × 15 = 15)**



8. (a) Solve : $\frac{dy}{dx} + y\tan x = \sin 2x$.

(b) Verify for exactness and solve

$(x^3 - 4xy - 2y^2)\,dx + (y^3 - 4xy - 2x^2)\,dy = 0$

(c) Find the general and singular solution of

$(y - px)(p - 1) = p$.

Or

9. (a) Solve : $\dfrac{dy}{dx} + \dfrac{1}{(1+x^2)} y = \dfrac{e^{\tan^{-1} x}}{(1+x^2)}$.

(b) Find the general and singular solution of $y = 3px + 6y^2 p^2$
(Hint : put $y^3 = V$ )

(c) Show that the family of curve

$$\frac{x^2}{\lambda} + \frac{y^2}{\lambda + 1} = 1 \text{ is self orthogonal.}$$