#### 61223

# Second Semester B.A./B.Sc. Degree Examination, May/June 2019

(CBCS – Freshers + Repeaters – 2014-15 and onwards)

#### **Mathematics**

# Paper II - MATHEMATICS - II

Time: 3 Hours] [Max. Marks: 70]

Instructions to Candidates : Answer all Parts.

#### PART - A

1. Answer any **FIVE** questions :

 $(5 \times 2 = 10)$ 

- (a) On the set z, \* is defined by a \* b = a + b 1,  $\forall a, b \in z$ . Find the identity element.
- (b) If in a group G,  $(ab)^2 = a^2b^2 \,\forall a, b \in G$ , then prove that G is abelian.
- (c) Find  $\phi$  for the curve  $r = a e^{\theta \cot \alpha}$ .
- (d) With usual notation prove that  $P = r \sin \phi$
- (e) Find the polar subtangent for the curve  $r = a(1 \cos \theta)$ .
- (f) Find  $\frac{ds}{dx}$  for the curve  $ay^2 = x^3$ .
- (g) Verify the exactness of the equation  $(e^y + 1)\cos x dx + e^y \sin x dy = 0$ .
- (h) Find the general solution for the equation  $y = px + \log p$ .

# PART – B

Answer **ONE** full question :

 $(1 \times 15 = 15)$ 

- If  $Q^+$  is the set of all positive rationals, prove that  $(Q^+, *)$  is an abelian 2. group where \* is defined by  $a * b = \frac{2ab}{3}$ .
  - Prove that the fourth roots of unity form an abelian group under (b) multiplication.
  - Prove that a non-empty subset H of a group (G, \*) is a subgroup of G if and only if

$$a * b^{-1} \in H, \forall a, b \in H$$

Or

- 3. (a) Prove that the inverse of an element in a group is unique.
  - Prove that  $H = \{0, 2, 4\}$  is a subgroup of a group  $G = \{0, 1, 2, 3, 4, 5\}$  under addition modulo 6.
  - Find: (c)

    - (ii)  $g \circ f$ , for the set  $A = \{1, 2, 3\}$  where  $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$  and  $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ .

PART - C

Answer any TWO full questions:

 $(2 \times 15 = 30)$ 

- With usual notations, prove that  $\tan \phi = r \frac{d\theta}{dr}$  for the polar curve,  $r = f(\theta)$ . 4. (a)
  - Show that the curves (b)  $r^n = a^n \cos n\theta$ ,  $r^n = b^n \sin n\theta$  intersect orthogonally.
  - Find the radius of curvature of the curve  $XY = C^2$ . (c)



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- 5. (a) Derive the formula for Radius of curvature in Cartesian form.
  - (b) Find the angle of intersection of the curves  $r = a(1 \cos \theta)$ ,  $r = 2a \cos \theta$ .
  - (c) Find the envelope of the family of lines  $\frac{x}{a} + \frac{y}{b} = 1$ , where a and b are connected by the relation  $ab = c^2$ .
- 6. (a) Find all the asymptotes of the curve  $x^3 + 2x^2y + xy^2 x^2 xy + 2 = 0$ .
  - (b) Find the position and nature of the double points of the curve  $x^3 + 2x^2 + 2xy y^2 + 5x 2y = 0$
  - (c) Find the area bounded by the Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

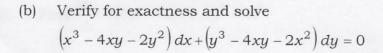
Or

- 7. (a) Find the pedal equation of the curve  $y^2 = 4a(x+a)$ .
  - (b) Find the perimeter of the cardioid  $r = a(1 + \cos \theta)$ .
  - (c) Find the volume of the solid generated by revolving the asteroid  $x^{2/3} + y^{2/3} = a^{2/3}$  about the x axis.

PART – D

Answer any **ONE** full question :

8. (a) Solve:  $\frac{dy}{dx} + y \tan x = \sin 2x$ .



(c) Find the general and singular solution of (y - px)(p-1) = p.

Or

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9. (a) Solve: 
$$\frac{dy}{dx} + \frac{1}{(1+x^2)}y = \frac{e^{\tan^{-1}x}}{(1+x^2)}$$
.

- (b) Find the general and singular solution of  $y = 3px + 6y^2p^2$ (Hint: put  $y^3 = V$ )
  - (c) Show that the family of curve  $\frac{x^2}{\lambda} + \frac{y^2}{\lambda + 1} = 1 \text{ is self orthogonal.}$

by re-ofting the asteroid

 $(1 \times 15 = 15)$ 



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