SS – 345

V Semester B.A./B.Sc. Examination, Nov./Dec. 2018
(Semester Scheme)
(Fresh + Repeaters) (CBCS) (2016-17 and Onwards)
Mathematics
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

   a) In a ring $(R, +, \cdot)$, show that $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)\ \forall a, b \in R$.

   b) Define subring of a ring and give an example.

   c) Show that the set of even integers is an ideal of the ring of integers.

   d) Find the unit normal vector to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at $(3, 1, -4)$.

   e) If $\phi = 2x^3y^2z^4$, then find $\nabla\phi$.

   f) Write the Newton's divided difference interpolation formula.

   g) Evaluate $\Delta^{10}(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$.

   h) State the Trapezoidal rule for the integral $\int_a^b f(x)\,dx$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Prove that the intersection of any two subrings is a subring. Give an example to show that the union of two subrings of a ring need not be a subring.

   b) Prove that $(z_5, +_5, \times_5)$ is a ring w.r.t. $+_5$ and $\times_5$.

OR

3. a) Prove that every field is an integral domain.

   b) Show that the set of all real numbers of the form $a + b\sqrt{2}$, where a and b are integers is a ring w.r.to addition and multiplication.

P.T.O.

4. a) If $f : R \to R'$ be a homomorphism and onto then prove that f is one-one iff Ker $f = \{0\}$.

b) Prove that the set $S = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle/ a, b \in z\right\}$ of all 2×2 matrices is a left ideal of the ring R over Z. Also show that S is not a right ideal.

OR

5. a) State and prove fundamental theorem of homomorphism of rings.

b) Find all the principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ w.r.to $+_8$ and $\times_8$.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 4z^2$ at the point $(1, 1, -8)$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$.

OR

7. a) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a non-zero constant. Also deduce that r^n is harmonic if $n = -1$.

b) If the vector $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, then find a.

8. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function. Then prove that $\operatorname{div}(\phi\vec{F}) = \phi(\operatorname{div}\vec{F}) + \nabla\phi \cdot \vec{F}$.

b) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

OR

9. a) Prove that :

i) Curl $\vec{F}$ is solenoidal.

ii) Grad ϕ is irrotational.

b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

PART – D

Answer **two full** questions. (2×10=20)

10. a) By the separation of symbols, prove that

$$u_0 + \frac{u_1}{1!} + \frac{u_2 x^2}{2!} + \ldots \infty = e^x \left[u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \ldots \infty \right]$$

b) Obtain the function whose first difference is $6x^2 + 10x + 11$.

OR

11. a) From the following data find 'θ' at $x = 84$ using difference table.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

b) Express $3x^3 - 4x^2 + 3x - 11$ in factorial notation. Also express its successive differences in factorial notation.

12. a) Prepare divided difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920

b) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$, by using Simpson's $3/8^{th}$ rule.

OR

13. a) By using Lagrange interpolation formula find f(10) from the following data.

x	5	6	9	11
f(x)	12	13	14	16

b) Evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub intervals, by using Simpson's $1/3^{rd}$ rule.