S1811213

61123

First Semester B.A./B.Sc. Degree Examinations, December 2018

(CBCS Scheme – Freshers)

MATHEMATICS

Paper I

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer all Parts.

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.

(b) Find the value of λ for which the following system has a nontrivial solution :

$$2x - y + 2z = 0$$
$$3x + y - z = 0$$
$$\lambda x - 2y + z = 0$$

(c) Find the nth derivative of $\log 3x$.

(d) If $u = x^3 + x^2y + xy^2 + y^3$ find $\frac{\partial^2 u}{\partial x^2}$.

(e) Evaluate $\int_0^{\pi/2} \sin^7 x\, dx$.

(f) Evaluate $\int_0^{\pi/2} \sin^5 x \cdot \cos^2 x\, dx$.

(g) Find the angle between the line $\frac{x+1}{3} = \frac{x-5}{3} = \frac{z+5}{6}$ and the plane $2x + 3y + 5z + 6 = 0$.

(h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z - 16 = 0$.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by reducing to row reduced echelon form.

(b) Verify the system of equations

$x + 2y + 2z = 1,\ 2x + y + z = 2,\ 3x + 2y + 2z = 3,\ y + z = 0$

for consistency and hence solve.

(c) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$.

Or

3. (a) Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

(b) For what values of λ and μ the following system of equations

$x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \mu z = \lambda$

have (i) unique solution (ii) an infinite number of solutions (iii) no solution.

(c) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

PART – C

Answer **TWO** full questions : **(2 × 15 = 30)**

4. (a) Find the nth derivative of $e^{ax}\sin(bx + c)$.

(b) Find the nth derivative of $\dfrac{1}{6x^2 - 5x + 1}$.

(c) If $x = \sin t$ and $y = \sin pt$ show that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - p^2)y_n = 0$.

Or

5. (a) State and prove Leibnitz's theorem for nth derivative of product of two functions.

(b) If $z = \sin(ax + y) + \cos(ax - y)$, show that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

(c) If $u = x^2 - 2y$ and $v = x + y$ find $\frac{\partial(u, v)}{\partial(x, y)}$.

6. (a) Find the reduction formula for $\int \cos^n x\, dx$ and evaluate $\int_0^{\pi/2} \cos^n x\, dx$.

(b) Evaluate $\int_0^{\infty} \frac{dx}{(1 + x^2)^4}$.

(c) Evaluate $\int_0^1 \frac{x^{\alpha} - 1}{\log x} dx$, $\alpha > -1$ is a parameter.

Or

7. (a) Find the reduction formula for $\int \tan^n x$ and evaluate $\int_0^{\pi/4} \tan^n x\, dx$.

(b) Evaluate $\int_0^a \frac{x^3 (a^3 - x^3)}{\sqrt{a^2 - x^2}} dx$.

(c) Evaluate $\int_0^{\pi} x \sin^4 x \cdot \cos^2 x\, dx$.

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Find the equation of the plane coaxial with the planes $6x + 4y - 5z - 2 = 0$ and $x - 2y + 3z = 0$ and parallel with direction ratios (1, 3, 2).

(b) Verify the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ for coplanarity.

(c) Find the equation of the sphere which passes through the points (3, 0, 0), (0, –1, 0), (0, 0, –2) and having its centre on the plane $3x + 2y + 4z - 1 = 0$.

Or

9. (a) Find the equation of the plane bisecting the angle between the planes $7x + 4y + 4z + 3 = 0$ and $2x + y + 2z + 2 = 0$.

(b) Find the shortest distance between the lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$.

(c) Find the equation of the right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to (2, –3, 6).