US – 355

VI Semester B.A./B.Sc. Examination, May 2017
(Fresh) (CBCS) (2016-17 and Onwards) (Semester Scheme)
MATHEMATICS – VIII

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer* ***all*** *questions/Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Evaluate $\lim_{z \to e^{\frac{i\pi}{4}}} \left(\frac{z^2}{z^4 + z^2 + 1} \right)$.

   b) Show that $|z - (2 + 3i)| = 5$ represents a circle.

   c) Prove that $u = y^3 - 3x^2y$ is a harmonic function.

   d) Define cross ratio of four points.

   e) Show that $f(z) = \sin z$ is analytic.

   f) State Liouville's theorem.

   g) Find the real root of the equation $x^3 - x - 2 = 0$ over the interval (1.5, 2) upto two approximation by bisection method.

   h) Write Newton Raphson iterative formula.

PART – B

Answer **four full** questions : **(4×10=40)**

2. a) Show that the locus of $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ is a line through the origin.

   b) Show that necessary condition for a function $f(z) = u(x, y) + i\, v(x, y)$ to be analytic.

OR

3. a) Evaluate $\lim_{z \to 2e^{\frac{i\pi}{6}}} \left(\frac{z^2 - 4}{z^3 + z + 5} \right)$.

   b) Show that $f(z) = \cos z$ is analytic and hence show that $f'(z) = -\sin z$.

P.T.O.

4. a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$.

   b) Find the orthogonal trajectories of the families of curves $e^{-x} \cos y + xy = c$.

   OR

5. a) If $f(z) = u + iv$ is analytic function then show that $\left(\frac{\partial f(z)}{\partial x}\right)^2 + \left(\frac{\partial f(z)}{\partial y}\right)^2 = |f'(z)|^2$.

   b) Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.

6. a) Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$.

   b) State and prove Cauchy's inequality theorem.

   OR

7. a) Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is $|z + 1 - i| = 2$.

   b) If $f(z)$ is analytic inside and on a simple closed curve C and a is a point within C then prove that $f^n(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}} dz$.

8. a) Show that $w = \frac{1}{z}$ transform a circle to circle or to a straight line.

   b) Discuss the transformation $w = \sin z$.

   OR

9. a) Find the bilinear transformation which maps $z = \infty, i, 0$ onto $w = 0, i, \infty$ respectively.

   b) Show that the transformation $w = \frac{i - z}{1 + z}$ makes the x-axis of the Z-plane onto a circle $|w| = 1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.

PART – C

Answer **two full** questions. **(2×10=20)**

10. a) Using bisection method find a real root of $x^3 - 3x^2 + 1 = 0$ correct to three places of decimal.

    b) Use Newton-Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places.

OR

11. a) Solve the equation $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 7$ using Jacobi's iteration method to third approximation.

    b) Find the largest eigen value of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method.

12. a) Using Taylor's series method find y at $x = 0.2$ correct to four decimal points given $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$.

    b) Solve using Runge-Kutta method $\frac{dy}{dx} = x + y$ and $y(0) = 1$ for $x = 0(0.2)0.4$.

OR

13. a) Solve $\frac{dy}{dx} = x - y$ by Euler's modified method with $y(0) = 1$ for $x = 0.2$ correct to 4 places of decimals.

    b) Using Euler's method solve $\frac{dy}{dx} = x - y$ for $x = 0(0.1)0.5$ given $y = 1$ when $x = 0$. Verify with the exact solution.
