VI Semester B.A./B.Sc. Examination, May 2017
(Semester Scheme)
(Fresh) (CBCS) (2016-17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions : (5×2=10)

a) Define a vector space over a field.

b) For what value of K the vectors (1, 2, 3), (4, 5, 6) and (7, 8, K) are linearly dependent.

c) Find the matrix of the linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (3x - y, 2x + 4y, 5x - 6y)$ w.r.t. the standard basis.

d) Find the null space of the linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$.

e) Write scalar factors in cylindrical co-ordinate system.

f) Solve : $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$.

g) Form a partial differential equation by eliminating the arbitrary constants from $z = (x + a)(y + b)$.

h) Solve : $pq + p + q = 0$.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) A subset W of a vector space V(F) is a subspace of V(F) if and only if $C_1\alpha + C_2\beta \in W$, for $\alpha, \beta \in W$.

   b) Find the basis and dimension of the subspace spanned by (1, – 1, 0), (0, 3, 1), (1, 2, 1) and (2, 4, 2) in V_3 (R).

OR

3. a) A set of non-zero vectors $(\alpha_1, \alpha_2, ...\alpha_n)$ of vector space V(F) is linearly dependent if and only if one of vectors Say α_K $(2 \leq K \leq n)$ is expressed as a linear combination of its preceding ones.

   b) Prove that $W = \{(x, y, z) \mid x = y = z\}$ is a subspace of R^3.

4. a) If $T : U \rightarrow V$ is a linear transformation then prove that

      i) $T(0) = 0'$ where 0 and $0'$ are the zero vectors of U and V respectively.

      ii) $T(-\alpha) = -T(\alpha), \forall \alpha \in U$.

   b) Find the linear transformation $T : R^2 \rightarrow R^2$ such that T (1, 0) = (1, 1) and T (0, 1) = (– 1, 2).

OR

5. a) State and prove rank nullity theorem.

   b) Show that the linear transformation $T : R^3 \rightarrow R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$ and $T(e_3) = 3e_1 + 4e_3$ is non singular where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Verify the condition of integrability and solve : $2yzdx + zxdy - xy(1 + z)dz = 0$.

   b) Solve : $(y - z)p + (z - x)q = x - y$.

OR

7. a) Show that cylindrical coordinate system is orthogonal co-ordinate system.

b) Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical co-ordinate and find f_ρ, f_ϕ, f_z.

8. a) Solve : $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

b) Solve : $\dfrac{dx}{y+z} = \dfrac{dy}{z+x} = \dfrac{dz}{x+y}$.

OR

9. a) Express the vector $\vec{f} = 3y\hat{i} + 2z\hat{j} + x\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z.

b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar co-ordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) Form partial differential equation by eliminating arbitrary function $f(xy + z^2, x + y + z) = 0$.

b) Solve : $p^2 - q^2 = x - y$.

OR

11. a) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 12xy$.

b) Solve : $z^2(p^2 + q^2) = x^2 + y^2$, by using the transformation $u = \frac{z^2}{2}$.

12. a) Find the complete integral of $z = pq$ by using Charpit's method.

b) Solve : $\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 12\frac{\partial^2 z}{\partial y^2} = e^{x-y} + x^2$.

OR

13. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \operatorname{Sin}^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

b) Solve : $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subjected to the condition :

i) $u(0, t) = 0$, $u(1, t) = 0$ for all t

ii) $u(x;0) = x^2 - x$, $0 \le x \le 1$.