



NS – 311

V Semester B.A./B.Sc. Examination, November/December 2016  
(Semester Scheme)  
(Fresh) (CBCS) (2016 – 17 and Onwards)  
MATHEMATICS – VI

Time : 3 Hours

Max. Marks : 70

**Instruction :** Answer **all** questions.

PART – A

Answer **any five** questions.

(5×2 = 10)

1. a) Write Euler's equation when  $f$  is independent of  $y$ .
- b) Show that the functional  $I = \int_{x_1}^{x_2} (y^2 + x^2 y^1) dx$  assumes extreme values on the straight line  $y = x$ .
- c) Define geodesic on a surface.
- d) Evaluate  $\int_C (5x dx + y dy)$  where  $C$  is the curve,  $y = 2x^2$  from  $(0, 0)$  to  $(1, 2)$ .
- e) Evaluate  $\int_0^\pi \int_0^{\sin y} y dx dy$ .
- f) Evaluate  $\int_0^1 \int_0^x \int_0^z dy dz dx$ .
- g) Show that the area of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\pi ab$  using Green's theorem.
- h) Evaluate using Stoke's theorem  $\oint_C (yz dx + zx dy + xy dz)$  where  $C$  is the curve  $x^2 + y^2 = 1, z = y^2$ .

P.T.O.



## PART - B

Answer **two full** questions :

(2×10=20)

2. a) Prove that the necessary condition for the integral  $I = \int_{x_1}^{x_2} f(x, y, y') dx$  with

$$y(x_1) = y_1 \text{ and } y(x_2) = y_2 \text{ to be an extremum is } \frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0.$$

b) Find the geodesic on a plane.

OR

3. a) Show that the extremal of  $I = \int_{x_1}^{x_2} \sqrt{y(1+(y')^2)} dx$  is a parabola.

b) Find the extremal of the functional  $I = \int_0^1 \sqrt{1+(y')^2} dx$  with  $y(0) = 1$  and  $y(1) = 2$ .

4. a) Find the shape of a chain which hangs under gravity between two fixed points.

b) Find the extremal of the functional  $\int_0^1 [(y')^2 + x^2] dx$  subject to constraint

$$\int_0^1 y dx = 2 \text{ and having end conditions } y(0) = 0, y(1) = 1.$$

OR

5. a) Find the function  $y$  which makes the integral  $I = \int_{x_1}^{x_2} [y^2 + 4(y')^2] dx$  an extremum.

b) Find the extremal of the functional  $I = \int_0^{\pi} [(y')^2 - y^2] dx$  with  $y(0) = 0$  and

$$y(\pi) = 1 \text{ and subject to the constraint } \int_0^{\pi} y dx = 1.$$



PART - C

Answer **two full** questions :

(2×10=20)

6. a) Evaluate  $\int_C (x + y + z) ds$  where C is the line joining the points (0, 1, 0) and (1, 2, 3).

b) Evaluate  $\iint_A (4x^2 - y^2) dx dy$ , where A is the area bounded by the lines  $y = 0$ ,  $y = x$  and  $x = 1$ .

OR

7. a) Evaluate  $\int_0^\infty \int_0^\infty x e^{x^2/y} dx dy$ , by changing the order of integration.

b) Find the area bounded by the arc of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in first quadrant.

8. a) Evaluate  $\int_0^1 \int_0^{x^2} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$ .

b) Evaluate  $\iint_R \frac{x^2 y^2}{x^2 + y^2} dx dy$  using polar co-ordinates, where R is the annular region between the circles  $x^2 + y^2 = 2$  and  $x^2 + y^2 = 1$ .

OR

9. a) Find the volume bounded by the surface  $z = a^2 - x^2$  and the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $y = b$ .

b) If R is the region bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $x + y + z = 1$ ,

show that  $\iiint_R z dx dy dz = \frac{1}{24}$ .



## PART – D

Answer **two full** questions :

(2×10=20)

10. a) State and prove Gauss' Divergence Theorem.

b) Evaluate using Green's theorem for  $\oint_C [xy \, dx + yx^2 \, dy]$ , where C is the curve enclosing the region bounded by the curve  $y = x^2$  and the line  $y = x$ .

OR

11. a) Verify Green's theorem in the plane for  $\oint_C [(x^2 - xy^3) \, dx + (y^2 - 2xy) \, dy]$ , where C is the square with vertices (0, 0), (2, 0), (2, 2) and (0, 2).

b) Evaluate  $\iiint_S \vec{F} \cdot \hat{n} \, ds$  using divergence theorem where  $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$  and S is the closed surface bounded by planes  $z = 0$ ,  $z = 1$  and the cylinder  $x^2 + y^2 = 4$ .

12. a) Verify Stokes theorem for  $\vec{F} = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$ ; C is the boundary of the upper half of the surface of the sphere  $x^2 + y^2 + z^2 = 9$ .

b) Evaluate using Gauss' divergence theorem  $\iiint_S \vec{F} \cdot \hat{n} \, ds$ , where

$\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$  and S is the total surface of the rectangular parallelepiped bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $x = 1$ ,  $y = 2$ ,  $z = 3$ .

OR

13. a) Evaluate  $\oint_C \vec{F} \cdot d\vec{r}$ , using Stoke's theorem where  $\vec{F} = (y - z + 2)\hat{i} + (yz)\hat{j} - (xz)\hat{k}$  taken over the surface S of the cube  $0 \leq x \leq 2$ ,  $0 \leq y \leq 2$ ,  $0 \leq z \leq 2$ .

b) By using Green's theorem evaluate  $\oint_C [(3x - y) \, dx + (2x + y) \, dy]$  where C is the circle  $x^2 + y^2 = a^2$ .