



NS – 309

V Semester B.A./B.Sc. Examination, Nov./Dec. 2016
(Semester Scheme)
(Fresh) (CBCS) (2016-17 Onwards)
MATHEMATICS – V

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

Answer any five questions :

(5×2=10)

1. a) In a ring $(R, +, \cdot)$, prove that $(-a) \cdot (-b) = a \cdot b; \forall a, b \in R$.
- b) Define subring of a ring. Give an example.
- c) Give an example of
 - i) Commutative ring without unity
 - ii) Non-commutative ring with unity.
- d) Find the unit vector normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$.
- e) Find the divergence of $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.
- f) Prove that $E = e^{hD}$.
- g) Write Lagranges Interpolation Formula.

h) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{3}{8}$ rule.

where

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y = f(x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.



PART - B

Answer **two full** questions.

(2×10=20)

2. a) Show that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are
- $a \in S, b \in S \Rightarrow a - b \in S$
 - $a \in S, b \in S \Rightarrow ab \in S$.
- b) Prove that every field is an Integral Domain.

OR

3. a) Show that the set of all matrices of the form $\left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} / a, b \in R \right\}$ is a non-commutative ring without unity with respect to addition and multiplication of matrices.
- b) Fill all the principal ideals of a ring $R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. $+_6$ and \times_6 .
4. a) Prove that $(Z_7, +_7, \times_7)$ is a commutative ring with unity. Is it a Integral Domain ?
- b) State and prove fundamental theorem of homomorphism.

OR

5. a) Prove that a commutative ring with unity is a field if it has no proper ideals.
- b) Prove that the mapping $f : (Z, +, \times) \rightarrow (2Z, +, *)$ where $a * b = \frac{ab}{2}$ defined by $f(x) = 2x, \forall x \in Z$ is an isomorphism.

PART - C

Answer **two full** questions.

(2×10=20)

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of $2\hat{i} - \hat{j} + \hat{k}$.
- b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that
- $\nabla r^n = n r^{n-2} \vec{r}$
 - $\nabla \left(\frac{1}{r} \right) = \frac{-\vec{r}}{r^3}$.

OR



7. a) Show that the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ intersect orthogonally at the point $(1, -1, 2)$.

b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla^2 \left(\text{div} \left(\frac{\vec{r}}{r^2} \right) \right) = \frac{2}{r^4}$.

8. a) If $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$, find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$.

b) If ϕ is scalar point function and \vec{F} is vector point function then $\text{curl} (\phi \vec{F}) = \phi \text{curl} \vec{F} + (\text{grad} \phi) \times \vec{F}$.

OR

9. a) If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that \vec{F} is irrotational then find ϕ such that $\vec{F} = \nabla\phi$.

b) Prove that $\text{curl} (\text{curl} \vec{f}) = \text{grad} (\text{div} \vec{f}) - \nabla^2 \vec{f}$.

PART - D

Answer two full questions.

(2x10=20)

10. a) Find a cubic polynomial which takes the following data :

x	0	1	2	3
f(x)	1	2	1	10

b) Find $f(1.4)$ from the following data.

x	1	2	3	4	5
f(x)	1	8	27	64	125

using difference table.

OR

11. a) Evaluate $\Delta(e^{3x} \log 4x)$.

b) Find $f(7.5)$ from the following data.

x	7	8	9	10
f(x)	3	1	1	9

using difference table.



12. a) Using Newton's divided difference formula find $f(3)$ from the given data.

x	0	1	2	5
f(x)	2	3	12	147

- b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

OR

13. a) Using Lagrange's interpolation formula find $f(2)$ from the following data.

x	0	1	3	4
f(x)	5	6	50	105

- b) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^{0.6} e^{-x^2} dx$.