NS – 307

III Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme) (CBCS) (F + R)
(2015-16 and Onwards)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Write the order of the elements of the group of the cube roots of unity under multiplication.

   b) Find all the left cosets of the subgroup $H = \{0, 4, 8\}$ in $(Z_{12}, \oplus_{12})$.

   c) Define a convergent sequence with an example.

   d) Show that $\left\{\frac{3n+5}{2n+1}\right\}$ is monotonic decreasing sequence.

   e) Test the convergence of the series $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \sqrt{\frac{3}{8}} + \ldots\ldots\ldots\ldots\ldots$

   f) Prove that every differentiable function is a continuous function.

   g) State the Lagrange's mean value theorem.

   h) Evaluate : $\lim\limits_{x \to a} \frac{x^a - a^x}{x^x - a^a}$.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) In a group G, prove that $O(a) = O(a^{-1}), \forall a \in G$.

   b) Prove that every subgroup of a cyclic group is cyclic.

   c) State and prove Fermat's theorem in groups.

OR

P.T.O.

3. a) If 'a' is a generator of a cyclic group of order 10, find the number of generators and write them.

b) Prove that every group of order less than or equal to 5 is abelian.

c) If 'n' is any positive integer and 'a' is relatively prime to n, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

PART – C

Answer **two full** questions : **(2×15=30)**

4. a) If $\lim\limits_{n \to \infty} a_n = a$ and $\lim\limits_{n \to \infty} b_n = b$, prove that $\lim\limits_{n \to \infty} (a_n \cdot b_n) = a \cdot b$.

b) Discuss the nature of the sequence $\left\{ n^{\frac{1}{n}} \right\}$.

c) Test the convergence of the sequences :

i) $\left\{ \sqrt{n+1} - \sqrt{n} \right\}$

ii) $\left\{ \dfrac{n + (-1)^n}{n} \right\}$.

OR

5. a) Prove that every convergent sequence is bounded.

b) Discuss the convergence of the sequences :

i) $\left\{ \sqrt{n^2 + 1} - 1 \right\}$

ii) $\left\{ \dfrac{3 + 7 + 11 + \ldots\ldots + (4n - 1)}{2n^2 + 3n} \right\}$.

c) Find the limit of the sequence 0.5, 0.55, 0.555, ...

6. a) State and prove D'Alembert's Ratio Test for the series of positive terms.

b) Discuss the nature of the series : $\dfrac{1}{\sqrt{1} + \sqrt{2}} + \dfrac{1}{\sqrt{2} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{4}} + \ldots\ldots\ldots\ldots$

c) Sum to infinity of the series $1+2\left(\frac{1}{9}\right)+\frac{2\cdot 5}{1\cdot 2}\left(\frac{1}{81}\right)+\frac{2\cdot 5\cdot 8}{1\cdot 2\cdot 3}\left(\frac{1}{729}\right)+.....$

OR

7. a) State and prove Cauchy's root test for a series of positive terms.

b) Discuss the convergence of the series $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+..........$

c) Sum to infinity of the series :

$$1+\frac{2^2}{1!}+\frac{3^2}{2!}+\frac{4^2}{3!}+..........$$

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Prove that every continuous function over a closed interval is bounded.

b) Test the differentiability of $f(x)=\begin{cases} x^2, & \text{if } x \le 3 \\ 6x-9, & \text{if } x > 3, \end{cases}$ at $x = 3$.

c) Evaluate :

i) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$

ii) $\lim_{x\to \infty}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$.

OR

9. a) Prove that a function which is continuous in closed interval takes every value between its bounds at least once.

b) State and prove Rolle's theorem.

c) Expand $\log(1+\sin x)$ upto the term containing x^4 by using Maclaurin's series.

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